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QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

This department was established as a medium for the discussion of questions relating to the teaching of collegiate mathematics and to incidental difficulties encountered by investigators. The publication of such questions and of replies to them naturally stimulates interest on the part of others whose work may be touched by these or by similar questions and difficulties.

Interest in mathematics, however, is largely stimulated by the publication of papers and a reader often feels that he could add something of value by way of extension or brief comment, but fears that what he has to offer may seem scarcely worthy of presentation as a formal paper. If, at such a time, he felt that there was some suitable place for publishing his contribution, he would probably prepare and submit it. It might, indeed, happen that a considerable part of such contributions would not prove suitable for publication; but it might also happen that many of the articles would not only furnish much of value to others but also frequently lead to larger and more extended investigations on the part of the author.

It has been suggested that this department ought to be broad enough to cover such discussions, and, believing the suggestion to be a good one, we are, accordingly, changing the heading to "Questions and Discussions" and publishing below two discussions which are due to interest aroused by the publication of "A Geometrical Discussion of the Regular Inscribed Heptagon" by J. Q. McNatt, in the January, 1914, issue.

In assuming charge of the department the new editor wishes to thank the retiring editor, Professor Carmichael, for kindly assistance, and to express the hope that all friends of the department will continue their generous coöperation and support.

All correspondence relating to this department should hereafter be addressed to U. G. MITCHELL, 1313 Massachusetts St., Lawrence, Kansas.

DISCUSSIONS.

Relating to the inscribed Heptagon, Nonagon and Undecagon.

(Cf. Article by J. Q. McNatt, this MONTHLY, Vol. XXI, pp. 13–14.)

I. DISCUSSION BY CLIFFORD N. MILLS, Brookings, S. D.

Let ADE be an equilateral triangle inscribed in a circle of unit radius and suppose the arc subtended by the side AD to be divided into three equal parts at points B and C. Draw the diameter HE and the chords AB, BC and CD. Also let fall a perpendicular BM from B to side AD. Let $2a$ be the length of the side of the regular inscribed nonagon.

To compute $2a$ in terms of the radius as a unit.

From the figure it is readily seen that

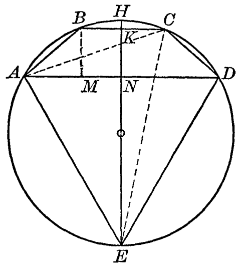
$$NE = 3/2, \qquad HN = 1/2, \quad \text{and} \quad \overline{BK}^2 = (3/2 + KN)(1/2 - KN).$$

But since $BK = a$ and $KN = \sqrt{3a^2 + a\sqrt{3} - 3/4}$

$$a^2 = 3/4 - \sqrt{3a^2 + a\sqrt{3} - 3/4} - 3a^2 - a\sqrt{3} + 3/4.$$

Simplifying gives

$$16a^4 + 8a^3\sqrt{3} - 12a^2 - 4a\sqrt{3} + 3 = 0.$$



Solving by Horner's method

$$a = .34202 \cdots,$$

which gives $2a = .68404\ldots$ as the side of the regular nonagon inscribed in a circle of radius unity.

Approximate Construction of a Nonagon.

By means of similar triangles it can be proved that $CD = CE - AC$ and that CD is approximately 2/5 of AD. Hence, if the side of an inscribed equilateral triangle be divided into five equal parts, a radius equal to the length of two of these parts is the approximate length of chord for inscribing a regular nonagon. In fact, the 2/5 of the side of the inscribed equilateral triangle is slightly less than 77/76 of the side of the inscribed regular nonagon.

II. Discussion by J. A. Colson, Searsport, Maine.

The "Heptagon Cubic" as given by Mr. J. Q. McNatt, in the January number of this Monthly, can be readily found by trigonometry from the following data:

$$h = 2 \sin \pi/7, \qquad \text{and} \qquad \sin \theta = \sin (\pi - \theta).$$

Put $\theta = 3\pi/7$, then $\pi - \theta = 4\pi/7$, and $\sin 3\pi/7 = \sin 4\pi/7$, or $3 \sin \pi/7 - 4 \sin^3 \pi/7 = (4 \sin \pi/7 - 8 \sin^3 \pi/7)\sqrt{(1 - \sin^2 \pi/7)}$.

Since $\sin \pi/7 \neq 0$, we may divide the above equation by $\sin \pi/7$, thus obtaining

$$3 - 4 \sin^2 \pi/7 = (4 - 8 \sin^2 \pi/7)\sqrt{(1 - \sin^2 \pi/7)}.$$

Square both members of this equation, unite similar terms, and we have

$$7 - 56 \sin^2 \pi/7 + 112 \sin^4 \pi/7 - 64 \sin^6 \pi/7 = 0.$$

But $\sin \pi/7 = h/2$. Hence by substitution, we obtain the "Heptagon Cubic"

$$7 - 14h^2 + 7h^4 - h^6 = 0.$$

This same equation can also be obtained by taking $\theta = 2\pi/7$ and $\pi - \theta = 5\pi/7$ or by taking $\theta = \pi/7$ and $\pi - \theta = 6\pi/7$ and proceeding in a manner similar to the above.

We may find the *Nonagon Cubic* by the same process. For, let n be a side of a regular nonagon inscribed in a circle whose radius is unity. Then $n = 2 \sin \pi/9$. Also $\sin \pi/3 = \sin (\pi - \pi/3) = \sin 2\pi/3 = 2 \sin \pi/3 \cos \pi/3$. Hence, dividing throughout by $\sin \pi/3$, we have

$$1 = 2 \cos \pi/3 = 2(1 - 4 \sin^2 \pi/9)\sqrt{(1 - \sin^2 \pi/9)}.$$

Square both members of this equation, unite similar terms, and we have

$$3 - 36 \sin^2 \pi/9 + 96 \sin^4 \pi/9 - 64 \sin^6 \pi/9 = 0.$$

Substitute for $\sin \pi/9$ its value $n/2$, and we have for the *Nonagon Cubic*

$$3 - 9n^2 + 6n^4 - n^6 = 0.$$

Hence, by Horner's Method, we have

$$n^2 = .4679111137620439295952\,15,$$

and finally

$$n = .6840402866513374660881\,99.$$

The *Undecagon Quintic* can be found in the same way. For let u be a side of a regular undecagon inscribed in a circle whose radius is unity. Then $u = 2 \sin \pi/11$.

Also $\sin 5\pi/11 = \sin (\pi - 5\pi/11) = \sin 6\pi/11$. Proceeding as before we find for the *Undecagon Quintic*

$$11 - 55u^2 + 77u^4 - 44u^6 + 11u^8 - u^{10} = 0.$$

Hence, by Horner's Method, we have

$$u^2 = .3174929343376376622763767,$$

and finally

$$u = .5634651136828593954228358.$$

Replies to Questions.

17. In analytic geometry, simplicity and directness are gained by making the condition for the collinearity of three points and the equation of the straight line depend upon the determinant formula for the area of a triangle. Similar advantages are gained by making the condition of complanarity of four points and the equation of the plane depend upon the determinant formula for the volume of a tetrahedron. The former is given in the texts. Why should not the latter be given? A uniform method of developing these two determinants is desired from some contributor.

Reply by A. M. Kenyon, Purdue University.

Let ABC be a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) and let A', B', C', be the projections on the x-axis of these points. We may suppose that the notation has been so chosen that $x_1 \leq x_2 \leq x_3$; and we assume at first that the vertices of the triangle all lie in the upper half plane.

$$\text{Area } ABC =^1 \pm [\text{Area } A'ACC' - \text{Area } A'ABB' - \text{Area } B'BCC']$$
$$= \pm \tfrac{1}{2}[(y_1 + y_3)(x_3 - x_1) - (y_1 + y_2)(x_2 - x_1) - (y_2 + y_3)(x_3 - x_2)]$$